Spiral cylindrique avec courbes terminales : 2 arcs de cercle et une droite

Poids du spiral et anisochronisme en position verticale

Déformations planes

Caractéristiques du spiral ***dextre***

Référence :E:\Résonateur (TA)\Data\Bal_spiral cylindrique (ex num).mcd(R)

Référence :E:\Résonateur (TA)\Data\Définition Atan.mcd(R)

Dimensions $\quad ép = 0.09\,mm \quad ha = 0.334\,mm \quad S = 0.03\,mm^2 \quad R_0 = 5\,mm \quad TOL := 10^{-9}$

Elinvar $\quad \rho_s = 8 \times 10^3\ kg \cdot m^{-3} \quad E = 1.7 \times 10^{11}\,Pa \quad G = 6.538 \times 10^{10}\,Pa$

Partie cylindrique $\quad n_s := 10.15 \quad \psi_0 := n_s \cdot 360 \cdot deg \quad \psi_0 = 3.654 \times 10^3\,deg \quad L := R_0 \cdot \psi_0 \quad L = 318.872\,mm$

$r_s(\alpha) := R_0 \quad s(\alpha) := R_0 \cdot (\alpha - \pi) \quad x_{0s}(\alpha) := R_0 \cdot cos(\alpha) \quad y_{0s}(\alpha) := R_0 \cdot sin(\alpha)$

Courbe terminale externe $\quad r_t := 0.5 \cdot R_0 \quad l_t := R_0 + \pi \cdot r_t \quad \alpha_A := \pi$

$x_{0t1}(\alpha_t) := r_t \cdot (1 + cos(\alpha_t)) \quad y_{0t1}(\alpha_t) := r_t \cdot sin(\alpha_t) \quad x_{0t2}(x) := x \quad y_{0t2}(x) := r_t$

$x_{0t3}(\beta_t) := -r_t \cdot (1 + sin(\beta_t)) \quad y_{0t3}(\beta_t) := r_t \cdot cos(\beta_t)$

Courbe terminale interne $\quad \alpha_B := mod(\psi_0 + \pi, 2 \cdot \pi) \quad \alpha_B = 234\,deg$

$x_{0t'1}(\alpha_{t'}) := x_{0t1}(\alpha_{t'}) \cdot cos(\alpha_B) - y_{0t1}(\alpha_{t'}) \cdot sin(\alpha_B) \quad x_{0t'2}(x) := x_{0t2}(x) \cdot cos(\alpha_B) - y_{0t2}(x) \cdot sin(\alpha_B)$

$y_{0t'1}(\alpha_{t'}) := x_{0t1}(\alpha_{t'}) \cdot sin(\alpha_B) + y_{0t1}(\alpha_{t'}) \cdot cos(\alpha_B) \quad y_{0t'2}(x) := x_{0t2}(x) \cdot sin(\alpha_B) + y_{0t2}(x) \cdot cos(\alpha_B)$

$x_{0t'3}(\beta_{t'}) := x_{0t3}(\beta_{t'}) \cdot cos(\alpha_B) - y_{0t3}(\beta_{t'}) \cdot sin(\alpha_B)$

$y_{0t'3}(\beta_{t'}) := x_{0t3}(\beta_{t'}) \cdot sin(\alpha_B) + y_{0t3}(\beta_{t'}) \cdot cos(\alpha_B) \quad L_t := 2 \cdot l_t + L$

Position du piton $\quad r_P := R_0 \quad \alpha_P := 0 \quad x_P := R_0 \quad y_P := 0 \cdot mm$

Position de la virole $\quad r_V := R_0 \quad \alpha_V(\theta) := mod(\alpha_B + \pi + \theta, 2 \cdot \pi) \quad \alpha_V(0) = 54\,deg$

$x_V(\theta) := r_V \cdot cos(\alpha_V(\theta)) \quad y_V(\theta) := r_V \cdot sin(\alpha_V(\theta))$

Amplitude stationnaire du balancier $\quad \theta_0 := 270 \cdot deg$

Moment quadratique de section

Référence :E:\Résonateur (TA)\Tables\Modules J, I et W des barres élastiques.mcd(R)

$I_{33} := I_{f\_rect}(ép, ha)$

Approximations de Haag

$\mathbf{OA} := R_0 \cdot e^{i \cdot \pi} \quad \mathbf{OB} := R_0 \cdot e^{i \cdot (\pi + \psi_0)}$

$X_{0t1}(\alpha_t) := r_t \cdot (1 + cos(\alpha_t)) \quad Y_{0t1}(\alpha_t) := r_t \cdot sin(\alpha_t) \quad X_{0t2}(x) := x \quad Y_{0t2}(x) := r_t$

$X_{0t3}(\beta_t) := -r_t \cdot (1 + sin(\beta_t)) \quad Y_{0t3}(\beta_t) := r_t \cdot cos(\beta_t)$

$$X_1 := \frac{1}{R_0^{\,2}} \cdot \left(\int_0^{\frac{\pi}{2}} X_{0t1}(\alpha) \cdot r_t \, d\alpha - \int_{r_t}^{-r_t} X_{0t2}(x)\, dx + \int_0^{\frac{\pi}{2}} X_{0t3}(\beta) \cdot r_t \, d\beta \right) \qquad X_1 = 0$$

$$Y_1 := \frac{1}{R_0^2}\cdot\left(\int_0^{\frac{\pi}{2}} Y_{0t1}(\alpha)\cdot r_t\,d\alpha - \int_{r_t}^{-r_t} Y_{0t2}(x)\,dx + \int_0^{\frac{\pi}{2}} Y_{0t3}(\beta)\cdot r_t\,d\beta\right) - 1 \qquad Y_1 = 0$$

$$\rho_1 := \sqrt{X_1^2 + Y_1^2} \qquad \varphi_1 := Atan(X_1, Y_1) \qquad \rho_1 = 0 \qquad \varphi_1 = 270\,deg$$

$$X_2 := \frac{1}{R_0^3}\cdot\left[\int_0^{\frac{\pi}{2}} r_t\cdot\alpha\cdot X_{0t1}(\alpha)\cdot r_t\,d\alpha - \int_{r_t}^{-r_t}\left(r_t\cdot\frac{\pi}{2} + r_t - x\right)\cdot X_{0t2}(x)\,dx + \int_0^{\frac{\pi}{2}}\left(r_t\cdot\frac{\pi}{2} + 2\cdot r_t + r_t\cdot\beta\right)\cdot X_{0t3}(\beta)\cdot r_t\,d\beta\right]$$

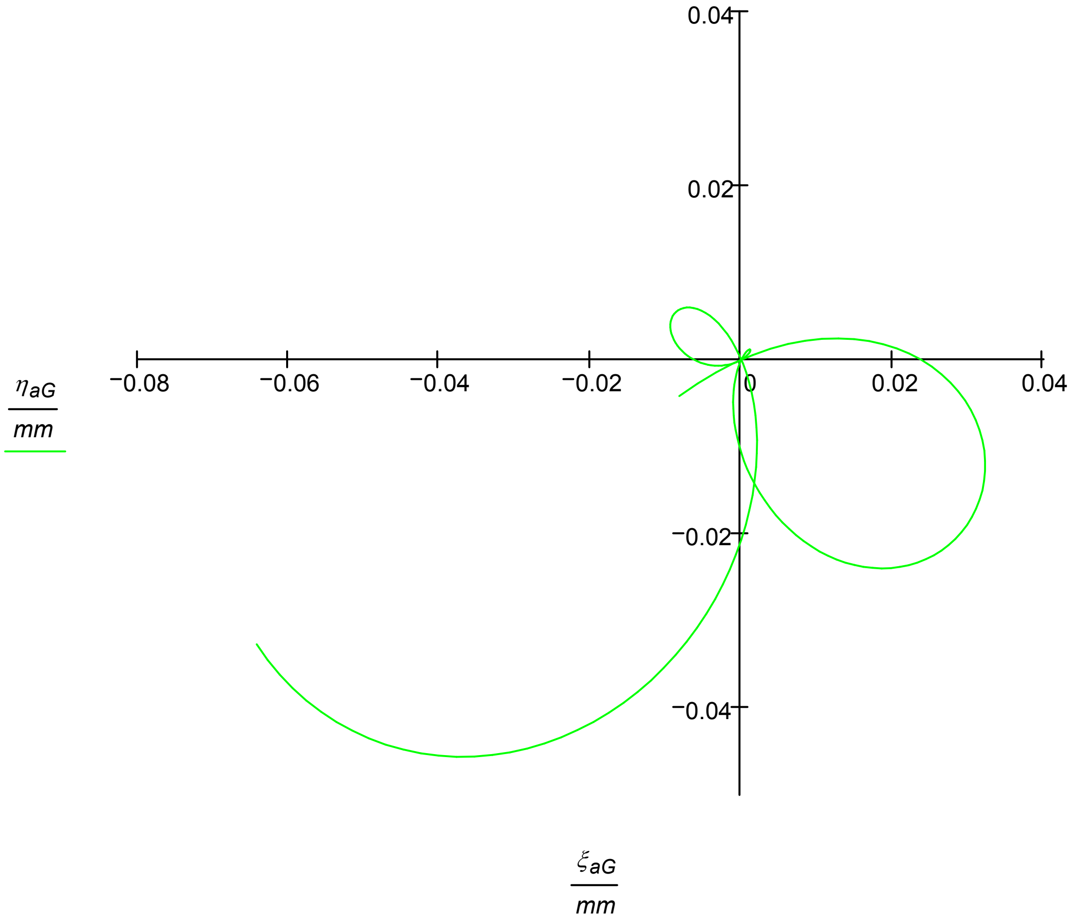
$$Y_2 := \frac{1}{R_0^3}\cdot\left[\int_0^{\frac{\pi}{2}} r_t\cdot\alpha\cdot Y_{0t1}(\alpha)\cdot r_t\,d\alpha - \int_{r_t}^{-r_t}\left(r_t\cdot\frac{\pi}{2} + r_t - x\right)\cdot Y_{0t2}(x)\,dx + \int_0^{\frac{\pi}{2}}\left(r_t\cdot\frac{\pi}{2} + 2\cdot r_t + r_t\cdot\beta\right)\cdot Y_{0t3}(\beta)\cdot r_t\,d\beta\right]$$

$$X_2 := X_2 + 1 \qquad \rho_2 := \sqrt{X_2^2 + Y_2^2} \qquad \varphi_2 := Atan(X_2, Y_2) \qquad \rho_2 = 1.316 \qquad \varphi_2 = 102.478\,deg$$

$$\omega(\theta) := \frac{\psi_0 + \theta}{2} + \varphi_2 \qquad \zeta_{aPh}(\theta) := -\frac{\theta}{\psi_0^2}\cdot\rho_2\cdot e^{-i\cdot\varphi_2}\cdot\boldsymbol{OA}\cdot e^{i\cdot\omega(\theta)}\cdot(\theta\cdot\cos(\omega(\theta)) + 4\cdot\sin(\omega(\theta)))$$

Graphes du déplacement du centre de gravité

$$n := 201 \qquad i := 0..n-1 \qquad \Delta\theta := \frac{4\cdot\pi}{n-1} \qquad \theta_i := -2\cdot\pi + i\cdot\Delta\theta \qquad \xi_{aG_i} := Re(\zeta_{aPh}(\theta_i)) \qquad \eta_{aG_i} := Im(\zeta_{aPh}(\theta_i))$$



Perturbation de période - spiral non déformé en position de repos

$$Q(\theta_0) := 5 \cdot J0(\theta_0) - \theta_0 \cdot J1(\theta_0)$$

$$Z_{aPh}(\theta_0) := \frac{-R_0^{\;2}}{2 \cdot L_t^{\;2}} \cdot \rho_2 \cdot \left(\mathbf{OA} \cdot e^{-i \cdot \varphi_2} + Q(\theta_0) \cdot \mathbf{OB} \cdot e^{i \cdot \varphi_2}\right) \qquad \delta_{aPh}(\theta_0) := -g \cdot \frac{m_s \cdot L}{E \cdot I_{33}} \cdot Im(Z_{aPh}(\theta_0))$$

$$\mu_{aPh}(\theta_0) := -86400 \cdot \delta_{aPh}(\theta_0) \qquad \boxed{\mu_{aPh}(\theta_0) = -4.107} \qquad \boxed{\mu_{aPh}(180 \cdot deg) = -8.156}$$

$$\theta_m := 60 \cdot deg, 65 \cdot deg .. 300 \cdot deg$$